

34. Perspective acceleration

ASTROMETRY CONCERNS the positions of objects on the sky, along with their projected motions in the plane of the sky due to their motion through space (their proper motion), and their apparent motion as the Earth orbits the Sun (their parallax).

Classical astrometry ignores a star's radial velocity, i.e. its space motion *along* the line-of-sight. This is because a star's radial velocity generally has no effect on its position on the sky. Indeed, not only does it have no effect on angular positional measurements, but conversely neither can its radial velocity be determined from these angular measurements. It is irrelevant in typical astrometric surveys, and it is ignored.

Knowing a star's radial velocity is nonetheless important for fully defining its complete space motion. The full space motions of stars are, in turn, essential in understanding kinematics and dynamics, both for individual stars, as well as for groups or populations.

Radial velocities are determined by measuring the Doppler shift of the stellar spectral lines. Extremely high accuracies can be reached in these velocity measurements: typical large-scale stellar surveys may reach accuracies of around 1 km s^{-1} , while today's dedicated high-precision radial velocity spectrometers used for exoplanet studies routinely measure stellar velocities, along the line-of-sight, with accuracies of about a few centimetres per second!

BUT RADIAL VELOCITIES are no longer irrelevant in very high accuracy astrometry, where it can affect the determination both of the parallax of a star, and of its proper motion. How is this possible?

This systematic change in trigonometric parallax due to the radial displacement of a star is most easily appreciated from the figure: a star moving through space, with some radial motion, has a parallax which changes with time, by a tiny amount proportional to the product of the radial velocity and the square of the parallax.

The effect is most apparent (if at all) for nearby stars with large radial velocities, but quickly diminishes for smaller radial velocities, and for larger distances.

The question was first considered by Schlesinger (1917). He concluded that the change in parallax is very small for all stars, and that detecting it would

need high-accuracy measurements over years or even decades. For Barnard's star, for example, with a parallax of about 500 mas, and a radial velocity of 110 km s^{-1} , the expected parallax change is a minuscule 34 microarcsec per year. It may just be measurable by Gaia.

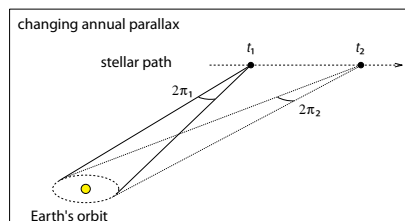
THE SECOND METHOD is somewhat similar, and exploits the fact that a single star moves with uniform velocity through space.

For a fixed space velocity, the angular velocity (or proper motion) varies inversely with the distance to the object. However, the tangential

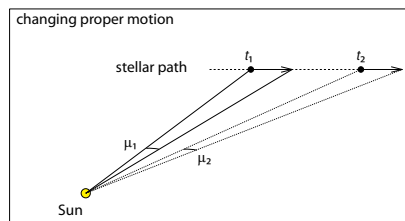
velocity changes due to the varying angle between the line-of-sight and the direction of its space velocity. The two effects result in a changing proper motion with time, which is interpreted as an apparent (or 'perspective') acceleration of the star's motion on the sky.

This apparent acceleration turns out to be proportional to the product of the star's parallax, its proper motion, and its radial velocity. It is always a tiny effect, but largest for nearby stars with a high proper motion.

Whether a changing proper motion is due to a *real* acceleration, e.g. if it is part of an orbital binary system, or due only to some unrecognised *perspective* acceleration, can only be clarified by further orbital measurements or by measurements of the star's radial velocity.



Drawins et al. (1999)



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HISTORICALLY, the phenomenon was first described by Schlesinger in 1901. It was used by Ristenpart (1902) in an attempt to determine a change in proper motion for Groombridge 1830, by Lundmark & Luyten (1922) for Barnard's star, and it was proposed by Russell & Atkinson (1931) as a way of confirming the hypothesised gravitational redshift, of several hundred km s^{-1} , predicted for the white dwarf Van Maanen 2.

In the 1970–1980s, photographic observations over several decades were used to measure this perspective acceleration for the white dwarf Van Maanen 2, for Groombridge 1830, and for Barnard's star.

Dravins et al. (1999) combined the Hipparcos results with those of the historical Astrographic Catalogue to determine astrometric radial velocities for 16 stars with large parallax–proper motion products. These included the fast moving Barnard's star, Kapteyn's star, as well as Groombridge 1830 and 61 Cygni.

Kürster et al. (2003) observed Barnard's star for more than five years with an accurate spectrograph at the ESO Very Large Telescope. They measured a secular acceleration fully consistent with the predicted value, of $4.50 \text{ m s}^{-1} \text{ yr}^{-1}$, based on the Hipparcos proper motion and parallax combined with the known radial velocity of $-110.506 \text{ km s}^{-1}$.

THE EFFORT INVESTED in this sort of task goes beyond attempting to measure the effect for its own sake. In practice, Doppler measurements represent the combination of the true velocity of the stellar centre of mass, combined with surface effects on the star such as atmospheric dynamics and gravitational redshifts.

The accurate determination of stellar radial velocities from geometric principles, i.e. without using spectroscopy or invoking the Doppler principle, can be used, in principle at least, both to examine these stellar phenomena, but also to establish fundamental radial velocity standards amongst the nearby stars.

AN EXTENSION of these principles applies to open clusters. Since all cluster stars share the same (average) velocity vector, apart from a (small) random velocity dispersion, the cluster's apparent size changes as it moves in the radial direction. This relative change, revealed by the proper motion vectors towards the cluster apex, corresponds to the relative change in distance. Since the individual stellar distances are known from parallaxes, their radial velocities can be estimated.

The method provides an estimate of the space velocity and internal velocity dispersion of a cluster using astrometric data only, along with improved parallaxes.

It was developed and applied to the Hipparcos data for the Hyades cluster by Lindegren et al. (2000) and to the Ursa Major, Coma Berenices, Pleiades, and Praesepe clusters by Madsen et al. (2002) with a number of detailed insights into the cluster members.

Predicted shifts (Δ in milliarcsec) for Gaia GDR2

Star name	Hipparcos catalogue	rad. vel. [km s^{-1}]	Δ [mas]
Barnard's star	87937	−110.51	1.975
Kapteyn's star	24186	245.19	1.694
Van Maanen 2	3829	263.00	0.573
61 Cyg A	104214	−65.74	0.313
61 Cyg B	104217	−64.07	0.297
Groombridge 1830	57939	−98.35	0.239
α Cen C (Proxima)	70890	−22.40	0.208
ϵ Ind	108870	−40.00	0.163
Ross 47	26857	105.83	0.144
ϵ Eri	15510	87.40	0.141

AT THE ACCURACY of Hipparcos, around 1 milli-arcsec, perspective acceleration was of marginal importance, and then only for the nearest stars with the largest radial velocities and proper motions. Nonetheless, it was accounted for in the 21 cases for which the accumulated positional effect over two years exceeds 0.1 milli-arcsec.

WHAT THEN of Gaia? For which stars is it most important, and what steps have been taken to include its effects? For the first Gaia data release, GDR1 in 2016, the astrometric accuracies were limited. The effect was simply ignored, by assuming zero radial velocity for all objects (Lindegren et al. 2016).

For the second data release in 2018, GDR2, it was taken into account for just 53 nearby Hipparcos catalogue objects, by using the values of radial velocities taken from the existing literature (Lindegren et al. 2018). The 10 stars with the highest predicted shifts over the 22 months of the Gaia data entering the construction of GDR2 are listed in the table, the largest being for Barnard's star, at 1.975 mas.

For the third data release, EDR3 in 2020, the effect was taken into account, whenever possible, using radial-velocity data from Gaia's own radial-velocity spectrometer (Lindegren et al. 2020). For a small number of nearby stars (mainly white dwarfs), this was complemented with radial velocities from the literature.

As an extension of its classical application to determine the distances to moving clusters, very clear perspective contraction has already been observed for the globular cluster NGC 3201, due to its very high radial velocity and relatively large parallax (Helmi et al. 2018).

GAI'A'S ACCURACIES will continue to improve over the coming years. And although perspective acceleration is not a dominant effect in the data, Gaia nonetheless provides a secure observational footing for the theoretical description of the effect presented more than a century ago. And it opens the possibility of some detailed investigations into the physics of stars for which the effect can be measured.