46. The iterative solution: formulation

GAIA GATHERS an enormous quantity of observations of a vast numbers of stars over several years. The goal of the data analysis on the ground is straightforward in principle: like solving a giant celestial jigsaw, the task is to find the positions and motions of each star best matching this gargantuan global set of observations.

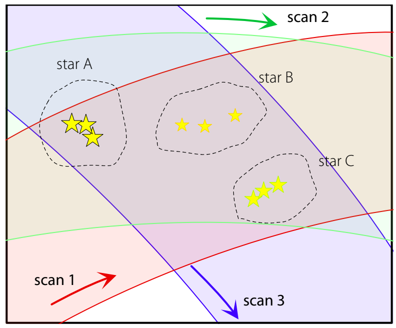
THE SCHEMATIC OPPOSITE shows three scans across a small region of sky to illustrate the concept. Depending on the scanning motion across that part of the sky at those particular times, the interval between successive scans may be several hours or several days.

Between the scans, all the stars have moved minutely, through a combination of their true motions through space, and their (apparent) parallax motions due to Earth's annual orbit around the Sun. Over months and especially over several years, and with many more scans, enough information has been collected to allow an estimate of each star's minuscule motion across the sky, along with any other motion that might affect it, such as orbital binary rotation, the effects of unseen planets, or gravitational light bending due to the Sun.

If you are looking at this problem for the first time, you may well be asking: How are the stars in the different scans matched up? What are the star positions measured with respect too? Why are there two fields of view? How are the optical aberrations of the telescope accounted for? How do any irregularities in the satellite's scanning motion affect the problem?

These considerations are all necessary for the correct and rigorous execution of the data processing (and are partly why these experiments take years to prepare and execute), but they are not central to the basic principles, and I will ignore most of them here.

AT THE HEART of the data processing carried out on the ground, then, is a global solution that matches up all the star signals generated by the CCD focal plane – several thousand every second – and solves for (a minimum of) the five astrometric parameters per star, along with all the additional unknowns describing any minute time-varying changes of the instrument.



The beauty of the problem is that the star positions, calibration and the spacecraft attitude are all tightly related, and connected by the fixed angle between the two identical telescopes simultaneously observing the sky.

And importantly, there is no satellite 'down time', in which science observations must be suspended while specific instrument calibrations are carried out; calibration is a by-product of the observations themselves.

But there is a catch: given the billions of stars, each with hundreds of observations, many thousands of calibration parameters, and with a satellite attitude sampled every second, any system of rigorous mathematical equations connecting all these unknowns is far too large and complex to solve directly.

The enormous size of the computational problem, and the experience gained through Gaia's predecessor, Hipparcos, led to the conclusion that only an iterative method might conceivably allow a solution.

INDEED, WHILE the *concept* is straightforward, the task of efficiently implementing and executing the global solution as an iterative least-squares adjustment was one of the major feasibility questions facing the Gaia project at the time of its adoption by ESA in 2000.

The mathematical solution to the problem was led by Lennart Lindegren, and described at the start of the mission by Lindegren et al. (2012), while Lindegren et al. (2016) addresses the details involved in the creation of Gaia DR1. I will look at some of the numbers involved in its numerical implementation separately.

GOING A LITTLE further into the problem, the challenge is the simultaneous estimation of a very large number of unknowns representing four distinct types of information: (a) the astrometric parameters for a subset of the observed stars, providing the astrometric reference frame; (b) the instrument attitude, representing the celestial pointing of the instrument axes in that reference frame as a function of time; (c) the geometric instrument calibration, representing the mapping from the CCD detectors to angular directions relative to the instrument axes; and (d) a few 'global' parameters describing, for example, a possible deviation of space–time from the prescriptions of general relativity.

Although the total number of stars observed by Gaia is more than two billion, only a subset are used in the astrometric core solution. This subset, of some 100 million well-behaved 'primary sources', consists of (effectively) single stars and extragalactic sources (quasars) that are sufficiently point-like and stable over time.

Nonetheless, the problem is formidable: the total number of unknowns involved is around a billion, and the solution uses some 100 billion observations extracted from some 100 000 Gbytes of raw satellite data.

AMONGST MANY details I will mention here just a few to give a flavour of the complexity.

The satellite 'attitude' specifies the telescope's orientation as it spins. The spacecraft is controlled to follow a specific 'scanning law', which provides good coverage of the entire sky, as well as maintaining a constant angle to the Sun. But the actual attitude can deviate from the nominal 'law' by up to 1 arcmin in all three axes.

The geometric instrument model defines the precise layout of the CCDs. It depends on their geometry, position and alignment in the focal-plane assembly, as well as the entire optical system including its scale, its stability, its distortions, and its other aberrations.

Gaia's high astrometric accuracy makes it necessary to use General Relativity to model the data. The formulation is based on the parametrised post-Newtonian (PPN) version of the relativistic framework adopted by the International Astronomical Union (IAU) in 2000.

Other complexities abound, including the chromaticity of the telescopes, charge transfer inefficiency of the CCDs, and attitude irregularities due to thruster noise and micro-meteoroid impacts.

THE NUMERICAL APPROACH to solving for all of these unknowns is a 'block iterative least-squares solution', the Astrometric Global Iterative Solution (AGIS).

In its simplest form, four 'blocks' are evaluated in a cyclic sequence until convergence. The blocks map to the four different kinds of unknowns mentioned previously: the source (star) update, S, in which the astrometric parameters of the primary sources are improved; the attitude update, A, in which the attitude parameters are improved; the calibration update, C, in which the calibration parameters are improved; the global update, G, in which the global parameters are improved.

The blocks must be iterated because each needs data from the three other processes. For example, when computing the astrometric parameters, the attitude, calibration and global parameters are taken from the previous iteration. These updated astrometric parameters are used the next time the A block is run. And so on!

WHILE THE BLOCK-ITERATIVE solution is intuitive and appealing in its simplicity, its implementation faced many challenges in practice: it is not obvious, mathematically, that it must converge. And if it does, it is not obvious how many iterations are required, whether the order of the blocks in each iteration matters, or even whether the converged results do, in fact, constitute a solution to the *global* minimisation problem.

Adding to the complexity is the fact that the core iterative solution also interfaces with all the other (enormous) processing tasks, amongst them the photometric analysis (including variability and 'alerts'), the treatment of double and multiple systems, the radial velocity measurements, and the object classification algorithms.

Accordingly, and in parallel with the industrial satellite development from about 2000 onwards, a Gaia 'Data Processing and Analysis Consortium' was set up with the task of developing and running a complete system to analyse all aspects of the satellite data, and so constructing the various Gaia catalogue releases.

AMONGST THE PEOPLE involved in this work (the co-authors of the 2012 papers were Uwe Lammers, David Hobbs, William O'Mullane, Uli Bastian, and José Hernández), Lennart Lindegren, of the Lund Observatory (Sweden), has made many and profound contributions to the Gaia project, and to Hipparcos before it, over a career of some 40 years.

In 2018 he was awarded the German Astronomical Society's *Instrumentation Prize* for his contributions to Gaia. And in 2020 he was awarded the Brouwer Award by the Division on Dynamical Astronomy of the American Astronomical Society, for his lifetime's contribution to astrometry.

The latter part of this citation reads *'His work has changed our understanding of the Universe at a fundamental level. . . The enduring legacy of his work is such that future generations of astronomers may owe their success, and even careers, to his remarkable contributions.*