150. Convection – and the mixing length

In essay #121, I looked at the 'big picture' of modelling stellar structure and evolution, pointing to some of the areas where Gaia is providing new constraints, and contributing to ongoing model refinements. Here, I want to focus on convection: why it is important, how it is modelled, and where Gaia is contributing.

In her review of the state of stellar evolution in the light of the Hipparcos results, Lebreton (2000) summarised the understanding of the energy transport processes in low- and intermediate-mass stars.

The size of a star's convective core, probed by asteroseismology, is important in the evolution of intermediate mass stars, as it defines the quantity of nuclear material available to drive the luminosity and lifetime. 'Overshooting' describes the penetration of convection and mixing beyond the classical Schwarzschild convection cores, and modifies the standard evolution model of stars of masses $M \gtrsim 1.2 M_\odot$, including their lifetimes.

Within the radiative zone, mixing processes include microscopic diffusion due to gravitational settling, which carries helium and heavy elements to the centre, affecting both evolution and surface abundances, while turbulent mixing (driven by rotation) inhibits it.

Numerical simulations are successful in reproducing the most important observational features of *solar* convection, such as high-resolution images, spectra, and helioseismology. But implementing convection effects in stellar evolution codes has proved challenging, essentially because the inherently 3-dimensional physical process can, most easily, only be treated as a one-dimensional phenomenological description.

In consequence, most stellar models are still built by treating convection according to the so-called 'mixing-length theory'. For the lower main sequence, radii are rather insensitive to the details of convection, and a solar-like tuning of the mixing-length parameter gives reasonable observational agreement. In contrast, the theory has limited predictive power for stars on the upper main sequence, and in other evolutionary phases.

A recent review of 'mixing length theory' has been given by Joyce & Tayar (2023). As they begin: *'The matter of energy transport in stars is notoriously complicated. In particular, the details of convection in the stellar interior are difficult to probe with direct observation and encompass the dominant sources of uncertainty in stellar models. Even the most specialised one-dimensional stellar evolution codes must simulate the behaviour of stars over enormous ranges in temperature, density, and pressure as well as over evolutionary timescales.'*

Convection thus represents one of the dominant sources of current modelling uncertainty, propagating through to substantial uncertainties in ages and, in turn, understanding of the chemical evolution of the Galaxy.

'Mixing length theory' describes the bulk movement of fluids within a convection zone, in analogy with molecular heat transfer. A hot 'parcel' of fluid, with locally uniform physical characteristics, and in pressure (but not thermal) equilibrium with its surroundings, will rise and expand towards cooler regions, and vice versa. The characteristic radial distance over which such a parcel can travel before losing its locally homogeneous physical characteristics can be considered as the mean-free path of that parcel, measured in terms of the pressure scale-height of the stratified fluid.

First applied to stellar interiors in an influential paper on solar convection by Böhm-Vitense (1958), mixing length theory remains the dominant framework for 1-d convective energy transport calculations. This longevity underlines not only its robustness and usefulness, but also the difficulty of constructing significantly better alternatives, even in today's era of computational power.

Popular codes for low- and intermediate-mass stars include ATON, BaSTI, CESAM, DSEP, GARSTEC, GENEC, MESA, Monash, PARSEC, STARS, and YREC.

Among these, only ATON and CESAM provide an alternative to mixing-length theory, viz. the full spectrum turbulence model (Canuto & Mazzitelli 1991; Canuto & Mazzitelli 1992; Canuto et al. 1996). More details on all of these aspects are given by Joyce & Tayar (2023).

Within this framework, the mixing length parameter, $\alpha_{\rm ML}$, is a dimensionless number characterising the distance that a parcel of convective material can travel, expressed in terms of the pressure scale-height. It can also be considered as a measure of convective efficiency: larger values of $\alpha_{\rm ML}$ imply that more thermal flux is carried by convection.

Main-sequence stars are typically modelled over three mass ranges: $M \lesssim 0.5_\odot$ (being fully convective); $M = 0.5 - 1.2 M_\odot$ (with radiative cores and convective envelopes); and $M \gtrsim 1.2 M_\odot$ (having convective cores and radiative envelopes). The choice of $\alpha_{\rm ML}$ is particularly important for stars with convective outer regions, $M \lesssim 1.2 M_\odot$. It can dramatically affect age estimates, since it affects the temperature structure, hence the nuclear burning rates, and therefore the main sequence lifetime. Stellar ages are, of course, a critical ingredient in many areas, from characterising exoplanet evolution to the enrichment and merger histories of galaxies.

As an example, a red giant's luminosity can be inferred from its parallax, and its temperature and metallicity from photometry and spectroscopy. Together, these provide the information needed to derive a model-dependent age. But because of the steepness of the red giant branch in the HR diagram, they can have very different inferred ages, by several Gyr, depending on the adopted mixing length. This will affect the inferred age–metallicity relation of the Galaxy, and any inferences of its merger and enrichment history (Freytag & Salaris, 1999). And similar problems arise in determining the age of globular clusters (e.g. Freytag & Salaris, 1999).

Testing models in this evolutionary regime has been difficult due to the lack of fundamental masses, and reliable spectroscopic measurements of temperature and detailed abundances, for large samples of stars. But Gaia's ability to provide accurate luminosities for very large numbers of red giants, covering a wide range of metallicity and age, has focussed renewed attention on the need to better characterise $\alpha_{\rm ML}$, and on developing improved models of its dependencies on $T_{\rm eff}$, $\log g$ and [Fe/H] (e.g. Tayar et al. 2017; Spada et al. 2021).

Based on asteroseismic quantities from LAMOST spectroscopy, Wang et al. (2023) derived masses and ages for 696 680 red giant branch stars (amongst others), drawing heavily on Gaia DR3 astrometry and photometry data to establish their location in the Hertzsprung–Russell diagram. With no specific consideration of the dependence on $\alpha_{\rm ML}$, their typical uncertainties were estimated as 10% on masses, and 30% on ages. Presumably these errors will be improved with a better modelling of the temperature and metallicity dependency of the mixing length. The large volume of the Galaxy covered by this sample ($5 < R < 20$ kpc and $|Z| < 5$ kpc) will make it of great value for a variety of Galactic studies.

In a very different field, Brandner et al. (2023b) used astrometric and photometric data from Gaia EDR3 to construct a sample of *bona fide* single stars in the Hyades cluster. The small observational errors result in a tightly defined stellar main sequence, particularly suitable for the testing and calibration of theoretical stellar evolutionary tracks and isochrones, in this case based on the MESA evolutionary models.

They found that the non-rotating MESA models for [Fe/H] = +0.25 provide a good fit for $M > 0.85 M_\odot$, and also for very low mass stars with $M \lesssim 0.25\,M_\odot$. For stars of intermediate mass, the models systematically underestimate the observed luminosity.

They concluded that a possible limitation of their models for stars above $0.35\,M_\odot$ is the prescription of (superadiabatic) convection with $\alpha_{\rm ML}$ tuned to match the solar model. This would have the consequence that the application of solar-scaled models to sub-solar mass stars could result in a significant underestimate of the age, or an overestimate of the metallicity.

Below 0.35 $\mathrm{M}_\odot$, they suggest that the increased scatter in the stellar sequence might be a manifestation of the 'convective kissing instability', predicted from evolutionary models by van Saders & Pinsonneault (2012).

Just above the threshold where main sequence stars are fully convective, around $0.35 M_\odot$, non-equilibrium ^{3}He burning creates a convective core, separated from a deep convective envelope by a small radiative zone. The steady increase in central ^{3}He causes the core to grow until it touches the surface convection zone, which triggers fully convective episodes in what they called the 'convective kissing instability'. These episodes lower the central abundance, and cause the star to return to a state in which it has a separate convective core and envelope. Importantly, this results in a few percent changes in radius and luminosity, and over Myr to Gyr timescales.

Similar work on model fitting to the tight main sequence of the Hyades cluster based on the recent Gaia astrometry and photometry, and its consequences for the inferred values of $\alpha_{\rm ML}$, are given by Brandner et al. (2023a) in the case of the PARSEC isochrones, and by Tognelli et al. (2021) in the case of the PISA models.

Let me finally mention a recent discussion of the theoretical dependencies of the mixing length affecting the classical Cepheid distance scale.

De Somma et al. (2022) examined the dependence of the period–radius and period–mass–radius relations for both the fundamental and first overtone modes. They confirmed the trend of a redder instability strip as metallicity increases. And they concluded that the combination of the mass–luminosity relation and $\alpha_{\rm ML}$ most consistent with the value found by Riess et al. (2021), in their determination of H_0, indeed corresponds to the canonical mass–luminosity relation with $\alpha_{\rm ML} = 1.7$.