225. Rotational parallaxes

Trigonometric parallaxes provide the fundamental geometric distance measurement principle at the heart of Gaia, and of Hipparcos before it. Distances are derived, essentially by straightforward trigonometry, from the difference in the angular position of a source, compared to background sources, along different lines-of-sight as the Earth (and Gaia) orbit the Sun.

There are other geometric distance methods, which only apply to specific source types, and which rest on radial velocities combined with angular measures. If there is a physical relation between the two, then the angular measures from astrometry (in $\mathrm{mas\,yr^{-1}}$) are related to the radial velocities (in $\mathrm{km\,s^{-1}}$) through the object's distance. Some examples will make this clearer:

(a) if the expanding shells of a supernova remnant are spherically symmetric, the maximum transverse motion (in the plane of the sky) is equal to the maximum radial velocity (on the line-of-sight). This 'expansion parallax' was applied to the Crab pulsar by Lundmark (1926) and Trimble (1973). Most recently, Lin et al. (2023) used radio VLBI to determine $\varpi = 0.53 \pm 0.06$ mas ($d = 1.90 \pm 0.20$ kpc), remarkably consistent with the parallax from Gaia DR3, $\varpi = 0.51 \pm 0.08$ mas (Antoniadis, 2020). It has also been used for planetary nebulae (e.g. Li et al., 2002).

(b) for pulsating Cepheids, the 'geometric' Baade–Wesselink method, or 'parallax-of-pulsation' method, relates the diameter changes during a pulsation cycle (using interferometry) to the radial velocity of its photosphere, yielding both its size and its distance (e.g. Lane et al., 2002; Breitfelder et al., 2016).

(c) orbital parallaxes provide distances for double-lined spectroscopic binaries with an astrometric orbit, again exploiting dual constraints from astrometry and radial velocities (e.g. Torres et al., 1997).

(d) kinematic distances to globular clusters can be obtained from a combination of proper motions and line-of-sight velocity measurements by appeal to simple spherical dynamical models, as first applied to M3 (Cudworth, 1979), and more recently ω Cen (van de Ven et al., 2006), 47 Tuc (McLaughlin et al., 2006), and others.

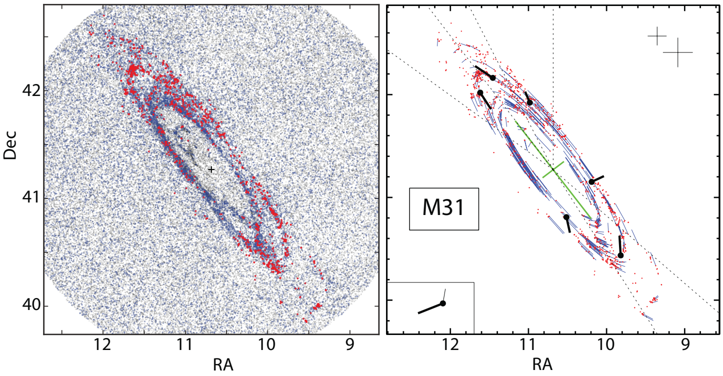
The 'rotational parallax' method of distance determination is in principle applicable to Local Group disk galaxies, such as M31 and M33, and it could provide an independent check of other extragalactic distance techniques (e.g. using Cepheids, RR Lyrae, tip of the red giant branch, red clump, Tully–Fisher, planetary nebulae, and eclipsing binaries).

As applied to M31 and M33, it was included as part of the extensive scientific case for Gaia in the Concept and Technology Study Report, ESA–SCI(2000)4. The Gaia data are not yet at the point that meaningful rotational parallaxes can be derived, but I will explain the principles, and describe the present status.

Consider a disk galaxy with known centre-of-mass space motion, flat rotation curve, i.e. independent of radial distance, and at some unknown inclination to the line-of-sight. Then for a target star on a circular orbit, three observables – its two proper motion components and its radial velocity – are, in principle, sufficient to determine the three unknowns: the orbit inclination, the rotational velocity, and the distance (Olling & Peterson, 2000; Olling, 2007). The method again makes use of the fact that since proper motions are distance-dependent, and radial velocities are distance-independent, determination of a rotation curve using both methods can provide the distance.

There are various complications in practice: the galaxy has some bulk space motion; stars are not on strictly circular orbits (e.g. due to spiral-arm streaming motions), they might have large z-heights or non-equilibrium velocities (e.g. runaway stars), and lie at significantly different distances (e.g. the near side of M31 is 5% closer than the far side). On the other hand, the galaxy's inclination can be estimated from the axis ratio of the image, or from the H I or Hα radial velocity fields.

Olling (2007) estimated that an accuracy of a few per cent in the distances to M31, M33 and the Large Magellanic Cloud requires radial velocities at the $10\,\mathrm{km\,s^{-1}}$ level, and proper motions which should be attainable by Gaia, or perhaps through VLBI observations of water masers in high-mass star-forming regions.



Van der Marel et al. (2019)

Left: the Gaia DR2 sources selected for M31, with blue and red points showing sources passing progressively stricter quality criteria. Right: the red points show the same sources, while the blue lines are the proper motions predicted by the best-fit rotating disk model. The inset shows the centre-of-mass proper motion (thick line), and the average proper motion of surrounding quasars (thin line). From van der Marel et al. (2019).

Another important reason for determining the rotational parallaxes of M31, M33, and others, is that the Local Group is the best and most proximate example of cosmological large-scale structure. Their orbits provide important tests of the predictions of hierarchical structure formation in the ΛCDM paradigm.

Such lofty goals are hampered by the small proper motions. As a result, their relative motions with respect to the Milky Way, central to understanding the past, present, and future evolution of the Local Group, have been subject to considerable debate (e.g. van der Marel et al., 2012; Banik et al., 2018; Patel et al., 2018; Semczuk et al., 2018; Salomon et al., 2021; Wempe et al., 2024).

For example, Patel et al. (2017) concluded that M33 is either on its first infall into the halo of M31, or that it is on a long-period (6 Gyr) orbit with a past pericentric approach at ~100 kpc. This would be consistent with first infall orbits expected for satellites in this mass range at the present epoch (Boylan-Kolchin et al., 2011).

Mapping of a galaxy's large-scale rotation based on 3D velocity measurements was first reported for the Large Magellanic Cloud by van der Marel & Kallivayalil (2014). They used HST proper motions in 22 fields (2–3 epochs over 2–7 yr), combined with line-of-sight velocities for 6790 stars. They applied the rotational parallax method by requiring that the rotation amplitude from the proper motions and radial velocities matched.

Complications included defining the galaxy's dynamical centre and the proper motion of its centre of mass, the inclination of its disk, and its rotation curve amplitude. They nonetheless derived a kinematic distance modulus for the LMC of 18.48 ± 0.40 mag, consistent but not competitive with other methods at the time.

Use of Gaia DR2 to study the proper motion fields of M31 and M33, providing many more stars than HST, and the next steps in determining their rotational parallaxes, was made by van der Marel et al. (2019). For the selection of member stars, they imposed cuts in parallax and proper motion, and in the relevant colour–magnitude diagrams.

The Gaia DR2 data do not meaningfully constrain the shape of the rotation curve, which they assumed to be flat (see Fig. 6 of Zhang et al., 2024). Nor do they yet reach the accuracy to significantly constrain the viewing angle, or the position, distance, and line-of-sight velocity to the centre-of-mass, for which they adopted existing values from photometric and radial velocity studies (with $D = 770$ kpc for M31, and 794 kpc for M33). The proper motion field is then determined by three free parameters, the two proper motion components of the centre of mass, and the rotation velocity of the disk.

Under various assumptions, they derived the proper motion of the centre-of-mass of both relative to background quasars in DR2, and the outward radial component in the proper motion field due to their approaching velocities. They detected the rotation of both galaxies (in the proper motion field), consistent with the known line-of-sight rotation curves: $V_{\rm rot} = -206 \pm 86\,{\rm km\,s^{-1}}$ (counter-clockwise) for M31, and $V_{\rm rot} = 80 \pm 52\,{\rm km\,s^{-1}}$ (clockwise) for M33. With future Gaia data releases, it should be possible to better quantify the outward radial component in the proper motion field, and obtain a kinematic distance estimate by comparison with the line-of-sight velocity of the centre-of-mass.

A similar analysis using Gaia DR3 was carried out by Rusterucci et al. (2024). They improved the connection to the quasar reference frame, and attributed variations in the inferred proper motions across different regions of M31 to systematics. They derived the bulk proper motion of M31 as $46.9 \pm 11.7 \pm 50.6\,\mu{\rm as\,yr^{-1}}$ in RA, and $-29.1 \pm 9.4 \pm 35.6\,\mu{\rm as\,yr^{-1}}$ in dec (statistical/systematics). They argued that the systematics remain the dominant source of uncertainty, being comparable to the proper motion of M31 itself. They provide equivalent results for M33.

They found that a significant reduction in the systematic uncertainties occurred between DR2 and DR3. They concluded that if similar progress is made with Gaia DR4, future Gaia-based estimates would match the level of uncertainties of HST, and could be used to refine the dynamics and history of both M31 and M33.