107.The white dwarf mass-radius relation

STELLAR EVOLUTION THEORY tells us that some 90% of stars will end their lives as white dwarfs, with different routes leading to the same fate. Theory further predicts that most white dwarfs, which are of intermediate mass, have an internal composition dominated by C or C–O cores, and a narrow mass distribution which peaks around $0.58 M_{\odot}$ (e.g. Bergeron et al., 1992).

The canonical mass for the ignition of He in the cores of red giants (the helium flash) is around $0.46 - 0.48 M_{\odot}$, so that white dwarfs with lower masses are inferred to have He cores. The latter are not believed to be a result of single star evolution since their progenitor's main-sequence lifetime exceeds the Hubble time. Instead, they are considered to have formed by Roche lobe overflow in a binary system, in which the low-mass He core is exposed as a result of mass transfer to the secondary before the progenitor reaches the tip of the red giant branch. For the high-mass tail, theory predicts interiors dominated by Ne and Mg (Panei et al., 2000).

As I have touched on in essay #29 (on their discovery with Gaia) white dwarfs span a vast temperature range, with the H-rich extending from 170 000 K to 4500 K. The classification scheme is according to the dominant spectral features: DA dwarfs have an atmosphere dominated by H I; DB are dominated by He I; DC have a continuous spectrum without deep lines; and so on.

FOR A fully-degenerate electron gas at densities of $10^6 - 10^8\,\mathrm{gm\,cm^{-3}}$, the equation-of-state leads to a mass–radius relation first derived by Chandrasekhar (1931). The relation was refined by Hamada & Salpeter (1961), who calculated fully degenerate white dwarf models of different chemical composition (He, C, Mg, Si, S, and Fe), and later by Wood (1990, 1995), and others.

The mass–radius relation remains a largely theoretical construct, but forms an underlying assumption in many related studies. It enters determination of their masses and their luminosity function, and hence a range of applications including distance calibration to globular clusters, the age of Galactic disk and halo by means of white dwarf cooling sequences, and constraints on variations of fundamental physical constants, notably $\dot{G}/G$.

EMPIRICAL CONFIRMATION of the mass–radius relation, and hence observational confirmation of stellar degeneracy, is challenging because of the few white dwarfs with accurately-determined masses and radii, and because their masses are concentrated in a rather narrow interval. Various approaches can be followed (e.g. Schmidt, 1996; Bédard, 2017):

(a) methods relating the stellar surface flux, H_λ, to the observed flux, f_λ ($f_\lambda = 4\pi(R^2/d^2)H_\lambda(T, \log g)$), where the solid angle R^2/d^2 depends on distance (d) and stellar radius (R). Surface fluxes are calculated from appropriate model atmospheres (and an assumed composition), using (say) values for $T_{\rm eff}$ and log g inferred either from high-resolution spectroscopy, or from synthetic photometry for those lacking spectral lines. Masses can then be derived by appeal to the mass–radius relation for the appropriate evolutionary models. The method is indirect in that is based on model atmospheres, but it is the only one that can be applied to field white dwarfs;

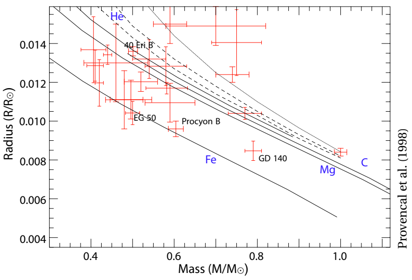
(b) the strong gravitational field at the white dwarf surface results in a spectral line redshift, $v_{\rm grs} = GM/Rc$. If $v_{\rm grs}$ can be measured, and the gravity is known, then M and R can be obtained independently of the parallax. $v_{\rm grs}$ can be measured in members of binary systems, common proper motion pairs, or clusters, for which the common component of the radial velocity allows separation of the gravitational redshift from the Doppler line shift due to the radial component of space motion;

(c) for white dwarfs in visual binary systems, masses can be derived from radial velocities and orbital parameters if the parallax is known (through Kepler's third law). Radii are then derived from $T_{\rm eff}$ and distance.

WHITE DWARFS are very common, but they have a very low intrinsic luminosity. As a result, reasonably bright objects are relatively nearby, such that even ground-based parallaxes, pre-Hipparcos, were already of reasonably high relative accuracy. Before Hipparcos, Schmidt (1996) argued that although masses and radii were consistent with theory, the observational scatter on both was too large to demonstrate a clear correlation.

HIPPARCOS OBSERVED just 22 white dwarfs (11 field, 4 in visual binaries, and 7 in common proper motion systems) among which the majority were of spectral type DA, but with one each of types DB, DC, DQ, DZ. All were close to the instrument's magnitude limit. The degraded mean accuracy, $\sigma_\pi \simeq 3.6$ mas, was nevertheless a factor 1.8 better than earlier ground-based values.

Several papers used these to re-examine the mass–radius relation (e.g. Vauclair et al., 1997; Shipman et al., 1997; Holberg et al., 1998; Provencal et al., 1998). In broad terms, the revised mass–radius relation was significantly narrower than the pre-Hipparcos situation, with most points within 1σ of evolutionary models of white dwarfs with C cores and H surface layers.



But the theoretical shape was still difficult to confirm, due to their small number, the absence of objects of high or low mass, and because of different possible evolutionary sequences. Small improvements, numerous discussions, and a still somewhat hazy final picture also accompanied the four objects in binary systems (Sirius, 40 Eri, Procyon, and Stein 2051).

Provencal et al. (1998) noted that Procyon B, and the field dwarfs GD 140 and EG 50, had radii much smaller than predicted by their masses inferred from C core models, and better matched Fe-rich core models (Isern et al., 1991). Debate surrounding this interpretation has continued (Panei et al., 2000a; 2000b), with extremely compact 'strange dwarfs' comprising quark matter, also being suggested (e.g. Mathews et al., 2006).

PROGRESS HAS continued to be made, in particular for these four binary systems. Figure 4 of Bond et al. (2017) shows the pre-Gaia status, including the mass determination for Stein 2051 derived from astrometric microlensing (Sahu et al., 2017). But, pre-Gaia, no truly semi-empirical test of the mass–radius relation for a single and isolated white dwarf had been made.

AND SO TO GAIA. In essay #29, I reviewed the numbers of white dwarfs that have been identified so far in Gaia DR2: some 260 000 in total, of which more than 8000 lie within 100 pc.

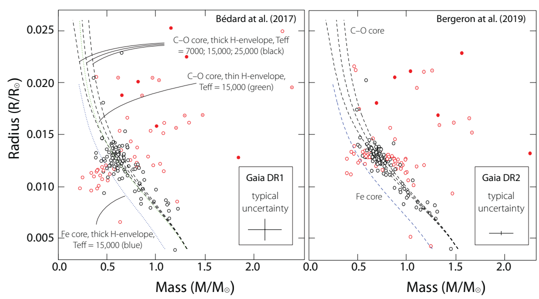
First, a single result: Gaia astrometry, and follow-up with HST, gave a mass for the isolated DQ white dwarf LAWD 37, exploiting an *astrometric* microlensing event (essay #11) in Nov 2019 (McGill et al., 2022). Gaia astrometry was used both for the event *prediction*, and the reference frame construction. The mass, $0.56 \pm 0.08 M_\odot$, is in agreement with the theoretical mass–radius relationship and cooling tracks for C–O core white dwarfs.

THREE STUDIES have used larger samples. Tremblay et al. (2017) used DR1 parallaxes for 6 isolated white dwarfs and 46 in wide binaries. Combined with spectroscopic atmospheric parameters, they found a good agreement between Gaia parallaxes, published $T_{\rm eff}$ and $\log(g)$, and theoretical mass–radius relations. But uncertainties on the atmospheric parameters dominated and, as for Hipparcos, the overall precision did not allow for the determination of H-envelope masses.

Bédard et al. (2017) used 206 DA and 13 DB white dwarfs with good parallaxes, 29 from DR1 (providing R), spectroscopy (providing $\log(g)$ and $T_{\rm eff}$), and photometry (providing the subtended solid angle), together allowing various tests of the mass–radius relation.

Their results are shown below (left). Of their 219 objects, 73% are consistent with the theoretical relation to within 1σ (black points), and so providing strong support for the theory of stellar degeneracy. They identified 15 stars that are better interpreted as unresolved double degenerate binaries (known double degenerates are shown as solid red; suspected as red circles with dots).

They also identified three objects that fall on the theoretical mass–radius relation for an *iron* core, their most compelling being G87–7 (also suggested by Provencal et al., 1998). Noting that the interpretation '*… definitely challenges the current theories of white dwarf formation*', it appears, however, that the Gaia DR2 parallaxes do *not* confirm this interpretation (Bergeron et al., 2019).



Gaia DR1 (Bédard et al. 2017); Gaia DR2 (Bergeron et al. 2019)

FINALLY, IN A paper which also describes various other inferences from the Gaia DR2 data, Bergeron et al. (2019) applied the 'photometric technique' to the determination of their radii, using DR2 parallaxes, and photometry from Pan-STARRS, SDSS, and Gaia.

Their updated mass–radius diagram for the same sources as Bédard et al. (2017), above right, shows the improvements with DR2. The parallaxes are, in fact, no longer the limiting error source, at least for the nearest.

A distinct result, worthy of mention here, is that the observed clump of low-mass, cool Gaia white dwarfs in the mass–$T_{\rm eff}$ diagram is, they conclude, evidence that the core composition of most is a mixture of carbon and oxygen… *'an expected result from stellar evolution theory, but never empirically well established before'.*